Published by Institute of Physics Publishing for SISSA/ISAS

Received: *April 15, 2002*
Revised: *May 31, 2002*
Accepted: *June 10, 2002*

Higher-order logarithmic conformal algebras from Robertson-Walker σ-model backgrounds

Elias Gravanis and Nick E. Mavromatos

Department of Physics, Theoretical Physics, King's College London,
Strand, London WC2R 2LS, United Kingdom
E-mail: eg@phpd88.ph.kcl.ac.uk, nm@phpd56.ph.kcl.ac.uk

Abstract: We demonstrate that an impulse action ('recoil') on a D-particle embedded in a (four-dimensional) cosmological Robertson-Walker (RW) spacetime is described, in a σ-model framework, by a suitably extended higher-order logarithmic world-sheet algebra of relevant deformations. We study in some detail the algebra of the appropriate two-point correlators, and discuss how can one approach the world-sheet renormalization group infrared fixed point, in the neighborhood of which the logarithmic algebra is valid. Interestingly enough it is found that, if the initial RW spacetime does not have cosmological horizons, then one can define a standard logarithmic algebra to describe the fixed point. However, in the presence of horizons, there are world-sheet divergences, due to relevant operators, which result in the presence of higher-order logarithmic algebras.

Keywords: Conformal Field Models in String Theory, D-branes, Sigma Models.

Contents

1. Introduction and summary

Placing D-branes in curved space times is not understood well at present. The main problem originates from the lack of knowledge of the complete dynamics of such solitonic objects. One would hope that such a knowledge would allow a proper study of the back reaction of such objects onto the surrounding space time geometry (distortion), and eventually a consistent discussion of their dynamics in curved spacetimes. Some modest steps towards an incorporation of curved space time effects in D-brane dynamics have been taken in the recent literature from a number of authors [1]. These works are dealing directly with world volume effects of D-branes and in some cases string dualities are used in order to discuss the effects of space time curvature.

A different approach has been adopted in [2]–[5], in which we have attempted to approach some aspects of the problem from a world sheet view point, which is probably suitable for a study of the effects of the (string) excitations of the heavy brane. We have concentrated mainly on heavy D-particles, embedded in a *flat* target background space time. We have discussed the instantaneous action (impulse) of a 'force' on a heavy D-particle. The impulse may be viewed either as a consequence of 'trapping' of a *macrosopic number* of closed string states on the defect, and their eventual splitting into pairs of open strings, or, in a different context, as the result of a more general phenomenon associated with the *sudden* appearance of such defects. Our world sheet approach is a valid approximation only if one looks at times *long after* the event. Such impulse approximations usually characterize classical phenomena. In our picture we view the whole process as a *semi-classical* phenomenon, due to the fact that the process involves open string *recoil* excitations of the heavy D-particle, which are *quantum* in nature. It is this point of view that we shall adopt in the present article.

Such an approach should be distinguished from the problem of studying single-string scattering of a D-particle with closed string states in flat space times [6]. We have shown in [2]–[5] that for a D-particle embedded in a d-dimensional *flat Minkowski* space time such an impulse action is described by a world-sheet σ-model deformed by appropriate 'recoil' operators, which obey a logarithmic conformal algebra [7]. The appearance of such algebras, which lie on the border line between conformal field theories and general renormalizable field theories in the two-dimensional world sheet, but can still be classified by conformal data, is associated with the fact that an impulse action (recoil) describes a *change* of the string/D-particle background, and as such it cannot be described by conformal symmetry all along. The *transition* between the two asymptotic states of the system before and (long) after the event is precisely described by deforming the associated σ-model by operators which *spoil* the conformal symmetry.

In this work we shall attempt to extend the flat space time results of [2, 3, 5] to the physically relevant case of a Robertson-Walker (RW) cosmological background space time. As well known in string σ-model perturbation theory, Robertson-Walker space times are not solutions of conformal invariance conditions of the σ-model, in the sense of having σ-model β-function different from zero. This would affect in general the two point correlators (c.f. below) $\langle X^\mu(z)X^\nu(w)\rangle$ which is modified from the standard $G^{\mu\nu}\ln|z-w|^2$ form by the inclusion of $\beta^{\mu\nu}$-dependent terms. Nevertheless, in the particular case of (large) cosmological times we are interested in here, which describe well the present era of the Universe, such terms are subleading, given that $\beta^{\mu\nu} \propto R^{\mu\nu} \sim 1/t^2$, and thus can be safely neglected. In this sense, discussing recoil in such (almost conformal) backgrounds is a physically interesting and non-trivial exercise in conformal field theory, which we would like to pursue here.

Although, our results do not depend on the target space dimension, however, for definiteness we shall concentrate on the case of a D-particle embedded in a four-dimensional RW spacetime. It must be stressed that we shall not attempt here to present a complete discussion of the associated space time curvature effects, which - as mentioned earlier - is a very difficult task, still unresolved. Nevertheless, by concentrating on times much larger than the moment of impulse on the D-particle defect, one may ignore such effects to a satisfactory approximation. As we shall see, our analysis produces results which look reasonable and are of sufficient interest to initiate further research.

The vertex operators which describe the impulse in curved RW backgrounds obey a suitably extended (higher-order) logarithmic algebra. The algebra is valid at, and in the neighborhood of, a non-trivial infrared fixed point of the world-sheet Renormalization Group. For a RW spacetime of scale factor of the form t^p, where t is the target time, and $p > 1$ in the horizon case, the algebra is actually a set of logarithmic algebras up to order $[2p]$, which are classified by the appropriate higher-order Jordan blocks [7].

As in the flat case, which is obtained as a special limit of this more general case, the recoil deformations are relevant operators from a world-sheet Renormalization-Group viewpoint. One distinguishes two cases. In the first, the initial RW spacetime does not possess cosmological horizons. In this case it is shown that the limit to the conformal world-sheet non-trivial (infrared) fixed point can be taken smoothly without problems and one has a standard logarithmic algebra. On the other hand, in the case where the initial spacetime has *cosmological horizons*, such a limit is plagued by world-sheet divergences. These should be carefully subtracted in order to allow for a smooth approach to the fixed point, leading to *higher-order* logarithmic algebras. We find this an interesting result which requires further study.

2. Recoiling D-particles in Robertson-Walker backgrounds

2.1 Geodesic paths and recoil

Let us consider a D-particle, located (for convenience) at the origin of the spatial coordinates of a four-dimensional space time, which at a time t_0 experiences an impulse. In a σ-model framework, the trajectory of the D-particle $y^i(t)$, i a spatial index, is described by inserting the following vertex operator

$$V = \int_{\partial\Sigma} G_{ij} y^j(t) \partial_n X^i \tag{2.1}$$

where G_{ij} denotes the spatial components of the metric, $\partial\Sigma$ denotes the world-sheet boundary, ∂_n is a normal world-sheet derivative, X^i are σ-model fields obeying Dirichlet boundary conditions on the world sheet, and t is a σ-model field obeying Neumann boundary conditions on the world sheet, whose zero mode is the target time.

This is the basic vertex deformation which we assume to describe the motion of a D-particle in a curved geometry to leading order at least, where spacetime back reaction and curvature effects are assumed weak. Such vertex deformations may be viewed as a generalization of the flat-target-space case [13].

Perhaps a formally more desirable approach towards the construction of the complete vertex operator would be to start from a T-dual (Neumann) picture, where the deformation (2.1) should correspond to a proper Wilson loop operator of an appropriate gauge vector field. Such loop operators are by construction independent of the background geometry. One can then pass onto the Dirichlet picture by a T-duality transformation viewed as a canonical transformation from a σ-model viewpoint [14]. In principle, such a procedure would yield a complete form of the vertex operator in the Dirichlet picture, describing the path of a D-particle in a curved geometry. Unfortunately, such a procedure is not free from ambiguities at a quantum level [14], which are still unresolved for general curved backgrounds. Therefore, for our purposes here, we shall consider the problem of writing a complete form for the operator (2.1) in a RW spacetime background in the Dirichlet picture as an open issue. Nevertheless, for RW backgrounds at large times, ignoring curvature effects proves to be a satisfactory approximation, and in such a case one may consider the vertex operator (2.1) as a sufficient description for the physical vertex operator of a D-particle. As we shall show below, the results of such analyses appear reasonable and interesting enough to encourage further studies along this direction.

For times long after the event, the trajectory $y^i(t)$ will be that of free motion in the curved space time under consideration. In the flat space time case, this trajectory was a straight line [13, 3, 4], and in the more general case here it will be simply the associated *geodesic*. Let us now determine its form, which will be essential in what follows.

JHEP06(2002)019

The space time assumes the form:

$$ds^2 = -dt^2 + a(t)^2(dX^i)^2 \tag{2.2}$$

where $a(t)$ is the RW scale factor. We shall work with expanding RW space times with scale factors

$$a(t) = a_0 t^p\,, \qquad p \in R^+ \tag{2.3}$$

The geodesic equations in this case read:

$$\begin{aligned} \ddot{t} + pt^{2p-1}(\dot{y}^i)^2 &= 0 \\ \ddot{y} + 2\frac{p}{t}(\dot{y}^i)\dot{t} &= 0 \end{aligned} \tag{2.4}$$

where the dot denotes differentiation with respect to the proper time τ of the D-particle.

With initial conditions $y^i(t_0) = 0$, and $dy^i/dt(t_0) \equiv v^i$, one easily finds that, for long times $t \gg t_0$ after the event, the solution acquires the form:

$$y^i(t) = \frac{v^i}{1-2p}\left(t^{1-2p}t_0^{2p} - t_0\right) + \mathcal{O}\left(t^{1-4p}\right), \qquad t \gg t_0 \tag{2.5}$$

To leading order in t, therefore, the appropriate vertex operator (2.1), describing the recoil of the D-particle, is:

$$V = \int_{\partial\Sigma} a_0^2 \frac{v^i}{1-2p}\Theta(t-t_0)\left(tt_0^{2p} - t_0 t^{2p}\right)\partial_n X^i \tag{2.6}$$

where $\Theta(t-t_0)$ is the Heaviside step function, expressing an instantaneous action (*impulse*) on the D-particle at $t = t_0$ [3, 5]. As we shall see later on, such deformed σ-models may be viewed as providing rather generic mathematical prototypes for models involving phase transitions at early stages of the Universe, leading effectively to time-varying speed of light. In the context of the present work, therefore, we shall be rather vague as far as the precise physical significance of the operator (2.6) is concerned, and merely exploit the consequences of such deformations for the expansion of the RW spacetime after time t_0, from both a mathematical and physical viewpoint.

In [3], we have studied the case $p = 0$, $a_0 = 1$, where the operators assumed the form $t\Theta_\epsilon(t)$ to leading order in t, where $\Theta_\epsilon(t)$ is the regulated form of the step function, given by [3]:

$$\Theta_\epsilon = -i\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\frac{1}{\omega - i\epsilon}e^{i\omega t}\,, \qquad \epsilon \to 0^+ \tag{2.7}$$

As discussed in that reference, this operator forms a logarithmic pair [7] with $\epsilon\Theta_\epsilon(t)$, expressing physically fluctuations in the initial position of the D-particle.

In the current case, one may expand the integrand of (2.6) in a Taylor series in powers of $(t-t_0)$, which implies the presence of a series of operators, of the form $(t-t_0)^q\Theta_\epsilon(t-t_0)$, where q takes on the values $2p, 2p-1, \ldots$, i.e. it is not an integer in general. In a direct generalization of the Fourier integral representation (2.7), we write in this case:

$$\begin{aligned} \mathcal{D}^{(q)} \equiv v_i(t-t_0)^q\Theta_\epsilon(t-t_0)\partial_n X^i &= v_i\; N_q \int_{-\infty}^{+\infty} d\omega \frac{1}{(\omega - i\epsilon)^{q+1}}\; e^{i\omega(t-t_0)}\partial_n X^i\,, \\ N_q \equiv \frac{i^q}{\Gamma(-q)(1-e^{-i2\pi q})} &= \frac{(-i)^{q+1}\Gamma(q+1)}{2\pi}\,, \end{aligned} \tag{2.8}$$

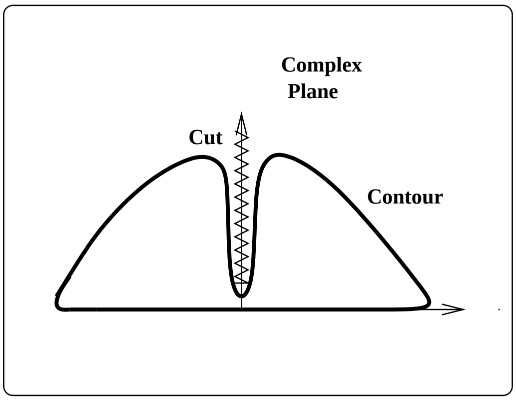

Figure 1: Contour of integration in the complex plane to define the recoil operators $\mathcal{D}^{(q)}$, by proper treatment of the associated cuts.

where we have incorporated the velocity coupling v_i in the definition of the σ-model deformation, and we have defined the integral along the contour of figure 1, having chosen the cut to be from $+i\epsilon$ to $+i\infty$.

2.2 Extended logarithmic world-sheet algebra of recoil in RW backgrounds

Following the flat space time analysis of [3], we now proceed to discuss the conformal structure of the recoil operators in RW backgrounds. We shall do so by acting on the operator $D^{(q)}$ (2.8) with the world-sheet energy momentum tensor operator $T_{zz} \equiv T$ (in a standard notation). Due to the form of the background space time (2.2), the stress tensor T assumes the form

$$2T = -(\partial t)^2 + a^2(t)(\partial X^i)^2 \tag{2.9}$$

where, from now on, $\partial \equiv \partial_z$, unless otherwise stated. One can then obtain the relevant operator-product expansions (OPE) of T with the operators $\mathcal{D}^{(q)}$. For convenience in what follows we shall consider the action of each of the two terms in (2.9) on the operators $\mathcal{D}^{(q)}$ separately. For the first (time t-dependent part), one has, as $z \to w$:

$$\begin{aligned} -\frac{1}{2}(\partial t(z))^2 \cdot \mathcal{D}^{(q)}(w) &= \frac{v_i}{(z-w)^2}\left[N_q \int_{-\infty}^{+\infty} d\omega \frac{\omega^2/2}{(\omega - i\epsilon)^{q+1}}\, e^{i\omega\, t(w)}\right]\partial_n X^i = \\ &= \frac{1}{(z-w)^2}\left[-\frac{\epsilon^2}{2}\mathcal{D}^{(q)} + q\epsilon\mathcal{D}^{(q-1)} + \frac{q(q-1)}{2}\mathcal{D}^{(q-2)}\right] \end{aligned} \tag{2.10}$$

The above formulæ were derived for asymptotically large time t, assuming the two-point correlators

$$\langle X^\mu(z)X^\nu(w)\rangle = 2G^{\mu\nu}\ln|z-w|^2 + \cdots, \tag{2.11}$$

where the $\cdots$ denote terms with negative powers of t, related to space-time curvature, which are subleading in the limit $t \to \infty$.

At this point, we stress again that Robertson-Walker space times are not solutions of conformal invariance conditions of the σ-model, as having β-functions different from zero. Non-zero β functions affect in general the two point correlators (2.11) by $\beta^{\mu\nu}$-dependent terms. In the particular case of (large) cosmological times, however, which describe well the present era of the Universe we are interested in here, such terms are subleading, given that $\beta^{\mu\nu} \propto R^{\mu\nu} \sim 1/t^2$, and thus can be safely neglected in the limit $t \to \infty$.

For the spatial part of (2.9) we consider the OPE $a(t(z))^2(\partial X^i(z))^2\mathcal{D}^{(q)}(w)$ as $z \to w$. Again, for convenience we shall do the time and space contractions separately:

$$t^{2p}(z)\cdot\mathcal{D}^{(q)}(w) = \int d\omega\tilde{\mathcal{D}}^{(q)}(\omega)t^{2p}(z)\cdot e^{i\omega(t(w)-t_0)} = \\ = \int_0^\infty \frac{d\nu}{\Gamma(-2p)}\nu^{-1-2p}\int d\omega\tilde{\mathcal{D}}^{(q)}(\omega)e^{-\nu t(z)}\cdot e^{i\omega(t(w)-t_0)} \tag{2.12}$$

Using the OPE $e^{-\nu t(z)}\cdot e^{-i\omega(t(w)-t_0)} \sim |z-w|^{i\nu\omega}e^{-\nu t(z)-i\omega(t(z)-t_0)+\mathcal{O}(z-w)}$ one obtains (as $z \sim w$):

$$t(z)^{2p}\cdot\mathcal{D}^{(q)}(w) = \int_0^\infty \frac{d\nu}{\Gamma(-2p)}\nu^{-1-2p}e^{-\nu t(z)}\mathcal{D}^{(q)}(t-t_0-\nu\ln|z-w|) = \tag{2.13}$$
$$= t^{2p}\int_0^\infty \frac{d\nu}{\Gamma(-2p)}\nu^{-1-2p}e^{-\nu}\mathcal{D}^{(q)}(t-t_0-\frac{\nu}{t}\ln|z-w|) =$$
$$= t^{2p}\left[\mathcal{D}^{(q)}(t-t_0) - \frac{1}{t}\ln|z-w|\frac{\Gamma(1-2p)}{\Gamma(-2p)}\frac{d}{dt}\mathcal{D}^{(q)}(t-t_0) + \mathcal{O}(t-t_0)^{q-2}\right].$$

We now observe that $\frac{d\mathcal{D}^{(q)}}{dt} = q\mathcal{D}^{(q-1)} - \epsilon\,\mathcal{D}^{(q)}$, where both terms have vacuum expectation values of the same order in ϵ, as we shall see below, and hence both should be kept in our perturbative expansion.

Expanding the various terms around t_0,

$$t^s = (t-t_0)^s + s\ t_0(t-t_0)^{s-1} + \frac{t_0^2}{2}(s)(s-1)(t-t_0)^{s-2} + \mathcal{O}([t-t_0]^{s-3}),$$

one has:

$$t^{2p}(z)\cdot\mathcal{D}^{(q)}(w) = \mathcal{D}^{(2p+q)}(t-t_0) + (2p\ t_0 - 2p\ \epsilon\ \ln|z-w|)\ \mathcal{D}^{(2p+q-1)} + \\ + \left(\frac{t_0^2}{2}2p(2p-1) + \left[2pq + (2p-4p^2)\epsilon\ t_0\right]\ln|z-w|\right)\mathcal{D}^{(2p+q-2)}(t-t_0) + \\ + \mathcal{O}\left([t-t_0]^{2p+q-3}\right) \tag{2.14}$$

where it is worthy of mentioning that inside the subleading terms there are higher logarithms of the form $\ln^n|z-w|$, where $n = 2, 3, 4, \ldots$

We now come to the OPE between the spatial parts. In view of (2.11), upon expressing ∂_z in normal ∂_n and tangential parts, and imposing Dirichlet boundary conditions on the world-sheet boundary where the operators live on, we observe that such operator products take the form:

$$(\partial X^j(z))^2\cdot\partial_n X^i(w) \sim G^{ii}\frac{1}{(z-w)^2}\partial_n X^i \sim \frac{t^{-2p}}{(z-w)^2}\partial_n X^i, \qquad \text{(no sum over } i) \tag{2.15}$$

Performing the last contraction with the t^{-2p}, following the previous general formulæ and collecting appropriate terms, one obtains:

$$
\begin{aligned}
T(z)\cdot\mathcal{D}^{(q)}[(t-t_0)(w)] = {} & \frac{1-\frac{\epsilon^2}{2}}{(z-w)^2}\mathcal{D}^{(q)}[(t-t_0)(w)] + \frac{q\epsilon}{(z-w)^2}\mathcal{D}^{q-1}[(t-t_0)(w)] + \\
& + \frac{\frac{q(q-1)}{2} - 2p^2\ln|z-w| - 2p^2\epsilon^2\ln^2|z-w|}{(z-w)^2}\mathcal{D}^{(q-2)}[(t-t_0)(w)] + \\
& + \mathcal{O}([t-t_0]^{q-3})
\end{aligned}
\tag{2.16}
$$

where again inside the subleading terms there are higher logarithms.

We next notice that, as a consistency check of the formalism, one can calculate the OPE (2.16) in case one considers matrix elements between *on-shell* physical states. In the context of σ-models, we are working with, the physical state condition implies the constraint of the vanishing of the world-sheet stress-energy tensor $2T = -(\partial t)^2 + a(t)^2(\partial X^i)^2 = 0$. This condition allows $(\partial X^i)^2$ to be expressed in terms of $(\partial t)^2$, which is consistent even at a correlation function level in the case of very target times $t \gg t_0$, since in that case, the correlator $\langle X^i t\rangle$ is subleading, as mentioned previously. Implementing this, it can be then seen that the OPE between the spatial parts of T and $\mathcal{D}^{(q)}$ is:

$$
\begin{aligned}
& a^2(t)(\partial X^i)^2\cdot\mathcal{D}^{(q)} = \\
& \quad = t^{-2p}(\partial t)^2\cdot\Big\{\mathcal{D}^{(2p+q)}(t-t_0) + (2pt_0 - 2p\epsilon\ln|z-w|)\,\mathcal{D}^{(2p+q-1)} + \\
& \qquad + \left(\frac{t_0^2}{2}2p(2p-1) + \left[2pq + \left(2p-4p^2\right)\epsilon t_0\right]\ln|z-w|\right)\mathcal{D}^{(2p+q-2)}(t-t_0) + \\
& \qquad + \mathcal{O}\left([t-t_0]^{2p+q-3}\right)\Big\}.
\end{aligned}
\tag{2.17}
$$

Performing the appropriate contractions, and adding to this result the OPE of the temporal part of T with $\mathcal{D}^{(q)}$, i.e. the quantity $-\frac{\epsilon^2}{2}\mathcal{D}^{(q)} + q\epsilon\mathcal{D}^{(q-1)} + \frac{1}{2}q(q-1)\mathcal{D}^{(q-2)}$, we obtain:

$$
\begin{aligned}
T\cdot\mathcal{D}^{(q)}|_{\text{on-shell}} = {} & \left(-2p\epsilon - pt_0\epsilon^2 + p\epsilon^2\ln\left(\frac{a}{L}\right)\right)\mathcal{D}^{(q-1)} + \\
& + \Big\{t_0^2\epsilon^2 2p(2p+1) - 3\epsilon^2 p(2p+q)\ln\left(\frac{a}{L}\right) - 2\epsilon^3\left(p+p^2\right)t_0\ln\left(\frac{a}{L}\right) - \\
& \quad - 2p^2\epsilon^4\ln^2\left(\frac{a}{L}\right) + \epsilon(2p+q)2pt_0 - \left(4p^2+4pq-2p\right)\Big\}\mathcal{D}^{(q-2)} + \\
& + \mathcal{O}\left([t-t_0]^{q-3}\right)
\end{aligned}
\tag{2.18}
$$

From the above we observe that the on-shell operators become marginal as they should, given that an on-shell theory ought to be conformal. Moreover, and more important, the world-sheet divergences *disappear* upon imposing the condition

$$
\epsilon^2\ln\left(\frac{L}{a}\right)^2 = \xi_0 = \text{constant independent of } \epsilon,\ a,\ L
\tag{2.19}
$$

where L (a) is the world-sheet (ultraviolet) infrared cut-off on the world sheet. As we shall discuss later on, this condition will be of importance for the closure of the logarithmic algebra, which characterizes the fixed point [3]. Hence, the conformal invariance is preserved by the on-shell states, any dependence from it being associated with *off-shell* states.

We next notice that, in the context of the RW metric (2.2), there are two cases of expanding universes, one corresponding to $0 < p \leq 1$, and the other to $p > 1$. Whenever $p \leq 1$ (which notably incorporates the cases of both radiation and matter dominated Universes) there is *no horizon*, given that the latter is given by:

$$\delta(t) = a(t) \int_{t_0}^{\infty} \frac{dt'}{a(t')} \tag{2.20}$$

In this case the relevant value for q is $q = 2p \leq 2$. On the other hand, for the case $p > 1$, i.e. $q > 2$ there is a non-trivial cosmological *horizon*, which as we shall see requires special treatment from a conformal symmetry viewpoint.

We commence with the no-horizon case, $1 < q \leq 2$. We first notice that the linear in t term in (2.6) leads to the conventional logarithmic algebra, discussed in [3], corresponding to a pair of impulse ('recoil') operators C, D. The main point of our discussion below is a study of the t^{2p} terms in (2.6), and their connection to other logarithmic algebras. Indeed, we observe that a logarithmic algebra [7, 2, 3] can be obtained for these terms of the operators, if we define $\mathcal{D} \equiv \mathcal{D}^{(q)}$ and $\mathcal{C} \equiv q\epsilon\mathcal{D}^{(q-1)}$. In this case we have the following OPE with T:

$$\begin{aligned}
(z-w)^2\, T \cdot \mathcal{D} &= \left(1 - \frac{\epsilon^2}{2}\right)\mathcal{D} + \mathcal{C}\,, \\
(z-w)^2\, T \cdot \mathcal{C} &= \left(1 - \frac{\epsilon^2}{2}\right)\mathcal{C} + \mathcal{O}\left([t-t_0]^{q-2}\right),
\end{aligned} \tag{2.21}$$

where throughout this work we ignore terms with negative powers in $t - t_0$ (e.g. of order $q-2$ and higher), for large $t \gg t_0$. Notice that in the case $q < 1$ (i.e. $p < 1/2$) the $\mathcal{C}$ operator defined above is absent.

In the second case $p > 1$ one faces the problem of having cosmological horizons (cf. (2.20)), which recently has attracted considerable attention in view of the impossibility of defining a consistent scattering S-matrix for asymptotic states [11, 12]. In this case the operator $\mathcal{D}^{(q-2)}$ is *not subleading* and one has an *extended (higher-order) logarithmic algebra* defined by (2.16). It is interesting to remark that now the logarithmic world-sheet terms in the coefficient of the $\mathcal{D}^{(q-2)}$ operator imply that the limit $z \to w$ is plagued by ultraviolet world-sheet divergences, and hence the world-sheet conformal invariance is spoiled. This necessitates Liouville dressing, in order to restore the conformal symmetry [8]. Such a dressing implies the presence of an extra space-time dimension given by the Liouville mode. The signature depends on the signature of the central charge deficit. We shall not deal with this procedure further in this article, the reason being that the RW background is itself *not conformal*.

We now turn to a study of the correlators of the various $\mathcal{D}^{(q)}$ operators, which will complete the study of the associated logarithmic algebras, in analogy with the flat target-space case of [3]. From the algebra (2.16) we observe that we need to evaluate correlators between $\mathcal{D}^{(q)}, \mathcal{D}^{(q-n)}$, $n = 0, 1, 2, \ldots$ We shall evaluate correlators $\langle \ldots \rangle$ with respect to the free world-sheet action, since we work to leading order in the (weak) coupling v_i. For convenience below we shall restrict ourselves only to the time-dependent part of the operators $\mathcal{D}$. The incorporation of the $\partial_n X^i$ is trivial, and will be implied in what follows. With these in mind one has:

$$\langle \mathcal{D}^{(q)}(z)\mathcal{D}^{(q-n)}(w)\rangle = N_q N_{q-n} \int\int_{-\infty}^{+\infty} \frac{d\omega d\omega'}{(\omega - i\epsilon)^{q+1}(\omega' - i\epsilon)^{q-n+1}} \langle e^{-i\omega t(z)} \; e^{-i\omega' t(w)}\rangle \tag{2.22}$$

where $\epsilon \to 0^+$. As already mentioned, we work to leading order in time $t \gg \infty$, and hence we can we apply the formula (2.11) for two-point correlators of the X^μ fields to write[1]

$$\begin{aligned} \langle e^{-i\omega t(z)} \; e^{-i\omega' t(w)}\rangle &= e^{-\frac{\omega^2}{2}\langle t(z)t(z)\rangle - \frac{\omega'^2}{2}\langle t(w)t(w)\rangle - \omega\omega'\langle t(z)t(w)\rangle} = \\ &= e^{-(\omega+\omega')^2 \ln(L/a)^2 + 2\omega\omega' \ln(|z-w|/a)^2}, \end{aligned} \tag{2.23}$$

where we took into account that $\mathrm{Lim}_{z\to w}\ \langle t(z)t(w)\rangle = -2\ln(a/L)^2$. Given that $\ln(L/a)$ is very large, one can approximate

$$e^{-(\omega+\omega')^2 \ln(L/a)^2} \simeq \frac{\sqrt{\pi}}{\sqrt{\ln(L/a)^2}}\delta(\omega + \omega')\,.$$

Thus we obtain:

$$\begin{aligned} \langle \mathcal{D}^{(q)}(z)\mathcal{D}^{(q-n)}(w)\rangle \;&=\; (-1)^{-q+n-1} N_q N_{q-n} \mathcal{J}_n^{(q)}\,, \\ \mathcal{J}_n^{(q)} \;&\equiv\; \sqrt{\frac{\pi}{\alpha}} \int_{-\infty}^{+\infty} \frac{d\omega e^{-\omega^2\lambda}(\omega + i\epsilon)^n}{(\omega^2 + \epsilon^2)^{q+1}} \end{aligned} \tag{2.24}$$

where $\lambda \equiv 2\ln(|z-w|/a)^2$, and $\alpha \equiv \ln(L/a)^2$.

Below, for definiteness, we shall be interested in the case $2 < q < 3$, in which the relevant correlators are given by $n = 0, 1, 2$. One has:

$$\begin{aligned} \mathcal{J}_0^{(q)} &= \sqrt{\frac{\pi}{\alpha}} \epsilon^{-2q-1} f_q\left(\epsilon^2\lambda\right)\,; \\ f_q(\xi) &= \sqrt{\pi}\frac{\Gamma(\frac{1}{2}+q)}{\Gamma(1+q)} F\left(\frac{1}{2}, \frac{1}{2} - q; \xi\right) + \xi^{\frac{1}{2}+q}\Gamma\left(-\frac{1}{2} - q\right) F\left(1 + q, \frac{3}{2} + q; \xi\right) \\ \mathcal{J}_1^{(q)} &= i\epsilon \mathcal{J}_0^{(q)}\,, \\ \mathcal{J}_2^{(q)} &= -2\epsilon^2 \mathcal{J}_0^{(q)} + \mathcal{J}_0^{(q-1)} = -\frac{\partial}{\partial\lambda}\mathcal{J}_0^{(q)} - \epsilon^2 \mathcal{J}_0^{(q)} \end{aligned} \tag{2.25}$$

where $F(a, b; z) = 1 + \frac{a}{b}\frac{z}{1!} + \frac{a(a+1)}{b(b+1)}\frac{z^2}{2!} + \cdots$ is the degenerate (confluent) hypergeometric function. Thus, the form of the algebra away from the fixed point ('*off-shell form*'), i.e.

[1] Here we use simplified propsagators on the boundary, with the latter represented by a straight line; this means that the arguments of the logarithms are real [3]. To be precise, one should use the full expression for the propagator on the disc, along the lines of [4]. As shown there, and can be checked here as well, the results are unaffected.

for $\epsilon^2 \neq 0$, is:

$$\begin{aligned}
\langle \mathcal{D}^{(q)}(z)\mathcal{D}^{(q)}(0)\rangle &= \\
&= \tilde{N}_q^2 \sqrt{\frac{\pi}{\xi_0}} \Bigg(f_q(2\xi_0)\left(\frac{\alpha}{\xi_0}\right)^q + 2f_q'(2\xi_0)\left(\frac{\alpha}{\xi_0}\right)^{q-1} \ln\left(\left|\frac{z}{L}\right|^2\right) + \\
&\qquad + \frac{1}{2} f_q''(2\xi_0)\left(\frac{\alpha}{\xi_0}\right)^{q-2} 4\ln^2\left(\left|\frac{z}{L}\right|^2\right) + \mathcal{O}\left(\alpha^{q-3}\right)\Bigg), \\
\epsilon q \langle \mathcal{D}^{(q)}(z)\mathcal{D}^{(q-1)}(0)\rangle &= \\
&= \tilde{N}_q^2 \sqrt{\frac{\pi}{\xi_0}} \Bigg(f_q(2\xi_0)\left(\frac{\alpha}{\xi_0}\right)^{q-1} + 2f_q'(2\xi_0)\left(\frac{\alpha}{\xi_0}\right)^{q-2} \ln\left(\left|\frac{z}{L}\right|^2\right) + \mathcal{O}\left(\alpha^{q-3}\right)\Bigg), \\
\epsilon^2 q^2 \langle \mathcal{D}^{(q-1)}(z)\mathcal{D}^{(q-1)}(0)\rangle &= \tilde{N}_q^2 \sqrt{\frac{\pi}{\xi_0}} f_{q-1}(2\xi_0)\left(\frac{\alpha}{\xi_0}\right)^{q-2} + \mathcal{O}\left(\alpha^{q-3}\right), \\
\epsilon^2 q(q-1) \langle \mathcal{D}^{(q)}(z)\mathcal{D}^{(q-2)}(0)\rangle &= -\tilde{N}_q^2 \sqrt{\frac{\pi}{\xi_0}} \left(f_q(2\xi_0) + f_q'(2\xi_0)\right)\left(\frac{\alpha}{\xi_0}\right)^{q-2} + \mathcal{O}\left(\alpha^{q-3}\right), \\
\epsilon^3 q^2(q-1) \langle \mathcal{D}^{(q-1)}(z)\mathcal{D}^{(q-2)}(0)\rangle &= \mathcal{O}\left(\alpha^{q-3}\right), \\
\epsilon^4 q^2(q-1)^2 \langle \mathcal{D}^{(q-2)}(z)\mathcal{D}^{(q-2)}(0)\rangle &= \mathcal{O}\left(\alpha^{q-4}\right)
\end{aligned} \tag{2.26}$$

where $\tilde{N}_q = \frac{\Gamma(1+q)}{2\pi}$, and ξ_0 has been defined in (2.19).

Notice that the above algebra is plagued by world-sheet ultraviolet divergences as $\epsilon^2 \to 0^+$, thereby making the approach to the fixed (conformal) point subtle. As becomes obvious from (2.19), the non-trivial fixed point $\epsilon \to 0^+$ corresponds to $L/a \to +\infty$, i.e. it is an infrared world-sheet fixed point. In order to understand the approach to the infrared fixed point, it is important to make a few remarks first, motivated by physical considerations.

From the integral expression of the regularized Heaviside function [3] (2.7) it becomes obvious that a scale $1/\epsilon$ for the target time is introduced. This, together with the fact that the scale ϵ is connected (2.19) to the world renormalization-group scales L/a, implies naturally the introduction of a 'renormalized' σ-model coupling/velocity $v_{R,i}(1/\epsilon)$ at the scale $1/\epsilon$:

$$v_{R,i}\left(\frac{1}{\epsilon}\right) \sim \left(\frac{1}{\epsilon}\right)^{q-1} \tag{2.27}$$

for a trajectory $y_i(t) \sim t^q$. This normalization would imply the following rescaling of the operators

$$\mathcal{D}^{(q-n)} \to \epsilon^{q-1}\mathcal{D}^{(q-n)} \tag{2.28}$$

As a consequence, the factors $\epsilon^{2(1-q)}$ in (2.25), (2.26) are removed. In the context of the world-sheet field theory this renormalization can be interpreted as a subtraction of the ultraviolet divergences by the addition of appropriate counterterms in the σ model.

The approach to the infrared fixed point $\epsilon \to 0^+$ can now be made by looking at the *connected* two point correlators between the operators $\mathcal{D}^{(q)}$ defined by

$$\langle \mathcal{A}\mathcal{B}\rangle_c = \langle \mathcal{A}\mathcal{B}\rangle - \langle \mathcal{A}\rangle\ \langle \mathcal{B}\rangle\,, \tag{2.29}$$

where the one-point functions are given by:

$$\langle \mathcal{D}^{(s)} \rangle = N_s \int \frac{d\omega}{(\omega - i\epsilon)^{s+1}} \langle e^{i\omega t} \rangle = N_s \int \frac{d\omega}{(\omega - i\epsilon)^{s+1}} e^{-\omega^2 \alpha} = \tilde{N}_s \epsilon^{-s} h_s \left(\epsilon^2 \alpha\right),$$

$$h_s(x) = -\frac{x^{s/2}}{2} \left(\frac{4\pi}{\Gamma(\frac{1+s}{2})} \sqrt{\pi} F \left(1 + \frac{s}{2}, \frac{3}{2}, x\right) - \frac{2\pi}{\Gamma(1 + \frac{s}{2})} F \left(\frac{1+s}{2}, \frac{1}{2}, x\right)\right). \quad (2.30)$$

For the two-point function of the $\mathcal{D}^{(q)}$ operator the result is:

$$\langle \mathcal{D}^{(q)}(z) \mathcal{D}^{(q)}(0) \rangle_c = \tilde{N}_q \epsilon^{-2} \left(\frac{\sqrt{\pi}}{\xi_0} f_q \left(2\xi_0 + 2\epsilon^2 \ln \left|\frac{z}{L}\right|^2\right) - h_q^2(\xi_0) \right). \quad (2.31)$$

Expanding in powers of ϵ, we obtain

$$\begin{aligned}\langle \mathcal{D}^{(q)}(z) \mathcal{D}^{(q)}(0) \rangle_c = {} & \tilde{N}_q \epsilon^{-2} \left(\frac{\sqrt{\pi}}{\sqrt{\xi_0}} f_q(2\xi_0) - h_q^2(\xi_0) \right) + \tilde{N}_q^2 \frac{\sqrt{\pi}}{\sqrt{\xi_0}} f_q'(2\xi_0) 2 \ln \left|\frac{z}{L}\right|^2 + \\ & + \epsilon^2 \tilde{N}_q^2 \sqrt{\frac{\pi}{\xi_0}} \frac{1}{2} f_q''(2\xi_0) 4 \ln^2 \left|\frac{z}{L}\right|^2 + \cdots \end{aligned} \quad (2.32)$$

where $\cdots$ denote terms that vanish as $\epsilon \to 0^+$.

To avoid the divergences coming from the ϵ^{-2} factors, the following condition must be satisfied: there must be a solution $\xi_0 = \xi_0(q)$ of the equation:

$$\mathcal{H}(\xi_0) \equiv \frac{\sqrt{\pi}}{\sqrt{\xi_0}} f_q(2\xi_0) - h_q^2(\xi_0) = 0\,.$$

The existence of such a solution can be verified numerically (see figure 2). Analytically this can be confirmed by looking at the asymptotic behaviour of the function $\mathcal{H}(x)$ as $x \to \infty$, which yields a negative value:

$$\mathcal{H}(x \to \infty) \sim -\frac{\pi^3 x^{2q} e^{2x}}{\Gamma^2(\frac{1+q}{2})\Gamma^2(1 + \frac{q}{2})} < 0\,.$$

This behaviour comes entirely from the term $h_q^2(x)$, given that $f_q(x \to \infty) \to 0^+$.

As we shall show below, for various values of q, near the fixed point $\epsilon \to 0^+$, one can construct higher order logarithmic algebras, whose highest power is determined by the dominant terms in the operator algebra of correlators (2.26), (2.21). To this end, we first remark that in the above analysis we have dealt with a small but otherwise arbitrary parameter ϵ, which allows us to keep as many powers as required by (2.26) in conjunction with the value of q. The value of ϵ determines the distance from the fixed point.

For $1 < q < 2$, there are only two dominant operators as the time $t \to \infty$, $\mathcal{D}$,$\mathcal{C}$. In this case one obtains a conventional logarithmic conformal algebra of two-point functions near the fixed point:

$$\begin{aligned}\langle \mathcal{D}^{(q)}(z) \mathcal{D}(0)^{(q)} \rangle_c &= \langle \mathcal{D}(z) \mathcal{D}(0) \rangle_c \sim \tilde{N}_q^2 \frac{\sqrt{\pi}}{\sqrt{\xi_0}} f_q'(2\xi_0) 2 \ln \left|\frac{z}{L}\right|^2, \\ \epsilon q \langle \mathcal{D}^{(q-1)}(z) \mathcal{D}^{(q)}(0) \rangle_c &= \langle \mathcal{C}(z) \mathcal{D}(0) \rangle_c \sim \tilde{N}_q^2 \left(h_q^2(\xi_0) - h_{q-1} h_q(\xi_0)\right), \end{aligned} \quad (2.33)$$

and all the other correlators are subleading as $t \to \infty$.

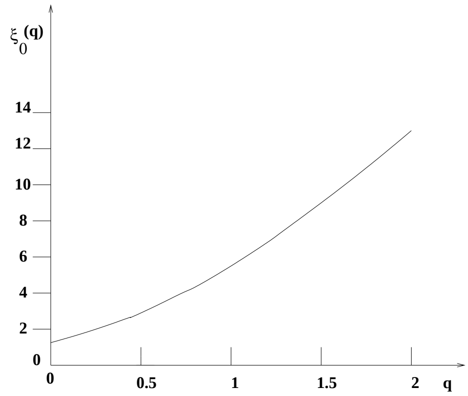

Figure 2: Graphic solution of the equation $\frac{\sqrt{\pi}}{\sqrt{\xi_0}} f_q(2\xi_0) - h_q^2(\xi_0) = 0$.

Therefore, the *on shell algebra* is of the conventional *logarithmic form* [7], between a pair of operators, and hence, $\mathcal{D}^{(q-2)}$ and subsequent operators, which owe their existence to the non-trivial RW metric, do not modify the two-point correlators of the standard logarithmic algebra of 'recoil' (impulse) [3].[2]

Next, we consider the case where $2 < q < 3$. In this case, from (2.26) we observe that there are now three operators which dominate in the limit $t \to \infty$, $\mathcal{D}$, $\mathcal{C}$ and $\mathcal{B} = \epsilon^2\, q\, (q-1)\mathcal{D}^{(q-2)}$, whose form is implied from (2.21), in analogy with $\mathcal{C}$. The corresponding algebra of correlators consists of parts forming a conventional logarithmic algebra, and parts forming a second-order logarithmic algebra, the latter being obtained from terms of order ϵ^2 in the appropriate two-point connected correlators (cf. (2.32) etc.), which are denoted by a superscript $\langle \cdots \rangle_c^{(2)}$:

$$\begin{aligned}
\langle \mathcal{D}(z)\mathcal{D}(0)\rangle_c^{(2)} &= \tilde{N}_q^2 \sqrt{\frac{\pi}{\xi_0}} \frac{1}{2} f_q''(2\xi_0) 4 \ln^2 \left|\frac{z}{L}\right|^2 , \\
\langle \mathcal{C}(z)\mathcal{D}(0)\rangle_c^{(2)} &= \tilde{N}_q^2 \sqrt{\frac{\pi}{\xi_0}} 2 f_q'(2\xi_0) \ln \left|\frac{z}{L}\right|^2 , \\
\langle \mathcal{C}(z)\mathcal{C}(0)\rangle_c^{(2)} &= \tilde{N}_q^2 \sqrt{\frac{\pi}{\xi_0}} f_{q-1}(2\xi_0) , \\
\langle \mathcal{B}(z)\mathcal{D}(0)\rangle_c^{(2)} &= -\tilde{N}_q^2 \sqrt{\frac{\pi}{\xi_0}} \left(f_q(2\xi_0) + f_q'(2\xi_0)\right) , \\
\langle \mathcal{C}(z)\mathcal{B}(0)\rangle_c^{(2)} &= \langle \mathcal{B}(z)\mathcal{B}(0)\rangle_c^{(2)} = 0
\end{aligned} \tag{2.34}$$

where the last two correlators are of order ϵ^4 and ϵ^6 respectively, that is of higher order than the ϵ^2 terms, and hence they are viewed as zero to the order we are working here.

An important relation in logarithmic conformal field theories is a "formal derivative" relation with respect to the anomalous dimension Δ, between the logarithmic set of operators [15]. In this respect, we mention that in the case of logarithmic algebras of order $[q]$

[2] We note at this stage that, in our case of non-trivial cosmological RW spacetimes, the pairs of operators $\mathcal{D}, \mathcal{C}$ do not represent velocity and position as in the flat space time case of ref. [3], but rather velocity and acceleration. This implies that, under a finite-size scaling of the world sheet, the induced transformations of these operators do not form a representation of the Galilean transformations of the flat-space-time case.

JHEP06(2002)019

we encounter here one has:

$$
\begin{aligned}
\frac{\partial \mathcal{C}}{\partial \Delta} &= q\mathcal{D} + \frac{\mathcal{C}}{2\Delta}\,, \\
\frac{\partial^2 \mathcal{B}}{\partial \Delta^2} &= q(q-1)\mathcal{D} + 3\frac{\mathcal{C}}{2\Delta}\,, \\
\cdots
\end{aligned}
\tag{2.35}
$$

where $\Delta = -\epsilon^2/2$, $\mathcal{C}$, $\mathcal{D}$ and $\mathcal{B}$ have been defined previously (c.f. (2.21)), and the ... denote similar relations for higher-order logarithmic algerbas than the ones examined in detail above, whose pattern can already be inferred easily. The first terms on the right-hand-side of these relations would be exactly the derivative relation of a standard logarithmic conformal field theory of order $[q]$ [15]. However in the recoil case one encounters singular $1/\sqrt{-\Delta} \sim 1/\epsilon$ terms due to the specific form of the operator $\mathcal{C}$. Such singular terms characterize also the corresponding derivative relations in the flat-space recoil case [3, 4] (but had not been discussed explicitly there[3]). It is worthy of stressing, though, that such singularities seem to characterise only the formal derivative relations and not the logarithmic O.P.E.'s or *the connected* correlators, as we have seen in detail above.

In general, if one considers $q > 3$ one arrives at higher order logarithmic algebras [7], with the highest power given by the integer value of q, $[q]$. This is an interesting feature of the recoil-induced motion of D-particles in RW backgrounds with scale factors $\sim t^p$, $p > 1$, corresponding to cosmological horizons and accelerating Universes. In such a case the order of the logarithmic algebra is given by $[2p]$. It is interesting to remark that radiation and matter (dust) dominated RW Universes would imply simple logarithmic algebras.

We now notice that, under a world-sheet finite-size scaling,

$$
L \to L' = Le^{\mathcal{T}\,\mathcal{K}(q)}\,, \qquad \epsilon^{-2} \to (\epsilon')^{-2} = \epsilon^{-2} + \mathcal{T}
\tag{2.36}
$$

with $\mathcal{K}(q)$ a function of q determined by (2.33), the operators $\mathcal{C}, \mathcal{D}, \ldots$, and consequently the target-time t, transform in a non trivial way. In particular, for t one has:

$$
\left(\frac{\epsilon'}{\epsilon}\right)^{q-1} \mathcal{Z}(\mathcal{T})^q t(\mathcal{T})^q = t^q + q\epsilon\mathcal{T}t^{q-1} + \mathcal{O}\left(\epsilon^2\right)
\tag{2.37}
$$

where $\mathcal{Z}(\mathcal{T})$ is a wave function renormalization of the world-sheet field $t(z)$, which can be chosen in a natural way so that $(\epsilon'/\epsilon)^{q-1}\mathcal{Z}\,(\mathcal{T})^q = 1$. This implies

$$
\begin{aligned}
t(\mathcal{T})^q &= (t + \epsilon\mathcal{T})^q + \mathcal{O}\left(\epsilon^2\right), \\
t(\mathcal{T}) &= t + \epsilon\mathcal{T} + \mathcal{O}\left(\epsilon^2\right),
\end{aligned}
\tag{2.38}
$$

i.e. that a shift in the target time is represented as $\epsilon\,\mathcal{T}$. Of course, at the fixed point, $\epsilon = 0$, the field $t(z)$ does not run, as expected.

[3] We thank the referee for alerting us to clarify this important point in the recoil cases examined here.

3. Vertex operator for the path and associated spacetime geometry

In this subsection we shall discuss the implications of the world-sheet deformation (2.1) for the spacetime geometry. In particular, we shall show that its rôle is to preserve the Dirichlet boundary conditions on the X^i by changing coordinate system, which is encoded in an induced change in the space time geometry G_{ij}. The final coordinates, then, are coordinates in the rest frame of the recoiling particle, which naturally explains the preservation of the Dirichlet boundary condition.

To this end, we first rewrite the world-sheet boundary vertex operator (2.1) as a bulk operator:

$$\begin{aligned} V &= \int_{\partial\Sigma} G_{ij} y^j(t) \partial_n X^i = \int_\Sigma \partial_\alpha \left(y_i(t) \partial^\alpha X^i \right) \\ &= \int_\Sigma \left(\dot{y}_i(t) \partial_\alpha t \partial^\alpha X^i + y_i \partial^2 X^i \right) \end{aligned} \tag{3.1}$$

where the dot denotes derivative with respect to the target time t, and α is a world-sheet index. Notice that it is the covariant vector y_i which appears in the formula, which incorporates the metric G_{ij}, $y_i = G_{ij} y^j$.

To determine the background geometry, which the string is moving in, it is sufficient to use the classical motion of the string, described by the world-sheet equations:

$$\partial^2 X^i + \Gamma^i{}_{\mu\nu} \partial_\alpha X^\mu \partial^\alpha X^\nu = 0\,, \tag{3.2}$$

where μ, ν are space time indices, $\alpha = 1, 2$ is a world-sheet index, ∂^2 is the laplacian on the world sheet, and i is a target spatial index.

The relevant Christoffel symbol in our RW background case, is $\Gamma^i{}_{ti}$, and thus the operator (3.1) becomes:

$$\int_\Sigma \left(\dot{y}_i - 2 y_i(t) \Gamma^i{}_{ti} \right) \partial_\alpha t \partial^\alpha X^i \tag{3.3}$$

from which we read an induced non-diagonal component for the space time metric

$$2G_{0i} = \dot{y}_i - 2 y_i(t) \Gamma^i{}_{ti} \tag{3.4}$$

In the RW background (2.2) the path $y_i(t)$ is described (2.5) by (notice again we work with covariant vector y_i):

$$y_i(t) = \frac{v_i a_0^2}{1 - 2p} \left(t t_0^{2p} - t_0 t^{2p} \right) \tag{3.5}$$

which gives $2G_{0i} = a^2(t_0) v_i$, yielding for the metric line element:

$$ds^2 = -dt^2 + v_i a^2(t_0) dt dX^i + a^2(t) (dX^i)^2\,, \qquad \text{for} \qquad t > t_0 \tag{3.6}$$

As expected, this spacetime has precisely the form corresponding to a Galilean-boosted frame (the D-particle's rest frame), with the boost occurring suddenly at time $t = t_0$.

This can be understood in a general fashion by first noting that (3.4) can be written in a general covariant form as:

$$2G_{0i} = \nabla_t y_i \quad (= \nabla_t y_i + \nabla_i t) \tag{3.7}$$

which is the general coordinate transformation associated with y_i from a passive (Lie derivative) point of view.

In general, given the boundary condition $\partial_n t = 0$, one can write the operator (2.1), in a covariant form by expressing it as a world-sheet bulk operator:

$$V = \int_{\partial\Sigma} y_\mu \partial_n X^\mu = \int_\Sigma \partial_\alpha \left(y_\mu \partial^\alpha X^\mu \right) = \int_\Sigma \nabla_\mu y_\nu \partial_\alpha X^\mu \partial^\alpha X^\nu \tag{3.8}$$

where in the last step, we have used again the string equations of motion (3.2). From this expression, one then derives the induced change in the metric

$$2\delta G_{\mu\nu} = \nabla_\mu y_\nu + \nabla_\nu y_\mu \tag{3.9}$$

which is the familiar expression of the Lie derivative under the coordinate transformation associated with y_μ.

In all the above expressions we have taken the limit $\epsilon \to 0$, which corresponds to considering the ratio of world-sheet cut-offs $a/L \to 0$, implying that one approaches the infrared fixed point in a Wilsonian sense. As noted previously, in the context of the logarithmic conformal analysis of the path $y^i(t)$, we have seen that this limit can be reached without problems only in the case $p \leq 1$, which corresponds to the absence of cosmological horizons. On the other hand, the case of non-trivial horizons, $p > 1$, implies ultraviolet divergences, which prevent one from taking this limit in a way consistent with conformal invariance of the underlying σ model. In such a case, the operators are relevant, with finite anomalous dimensions $-\epsilon^2/2$. One way to deal with such relevant operators is by Liouville dressing [8, 5] which would in principle restore the conformal symmetry at the cost of implying an extra target-space-time dimension. However in our case, such a restoration would not solve the full problem, since as we mentioned above we have neglected in our approach terms proportional to the graviton β-functions.

4. Conclusions

In this work we have analyzed in some detail the problem of impulse(recoil)-induced motion of a heavy D-particle in a Robertson-Walker spacetime, at large times t after the moment of impact. We have shown, that for RW spacetimes with scale factors $\sim t^p$, there is an order $[2p]$-logarithmic algebra, involving a group of impulse operators, which are relevant from a world-sheet renormalization group point of view.

A detailed study of how one can approach the non-trivial infrared fixed point is given. In the case where $p > 1$, which is physically characterized by the presence of cosmological horizons, one encounters world-sheet divergences. A proper subtraction of such divergences is subtle. The fact that away from the fixed point the deformed theory is plagued by

JHEP06(2002)019

relevant deformations, of anomalous dimension which itself depends on the world-sheet renormalization-group scale, implies the need for Liouville dressing. However, in our case, such a procedure cannot completely restore the conformal symmetry, in view of its violation by the very RW background.

Acknowledgments

We thank R. Szabo for discussions. The work of E.G. is supported by a King's College London Research Studentship (KRS).

References

[1] For some recent references see: J.E. Lidsey, *Dilatonic domain walls and curved intersecting branes*, *Phys. Rev.* **D 64** (2001) 063507 [hep-th/0106081];
M. Abou-Zeid, *Actions for curved branes*, hep-th/0001127;
G. Papadopoulos, J.G. Russo and A.A. Tseytlin, *Curved branes from string dualities*, *Class. and Quant. Grav.* **17** (2000) 1713 [hep-th/9911253];
B. Janssen, *Curved branes and cosmological (a,b)-models*, *J. High Energy Phys.* **01** (2000) 044 [hep-th/9910077].

[2] I.I. Kogan and N.E. Mavromatos, *World-sheet logarithmic operators and target space symmetries in string theory*, *Phys. Lett.* **B 375** (1996) 111 [hep-th/9512210].

[3] I.I. Kogan, N.E. Mavromatos and J.F. Wheater, *D-brane recoil and logarithmic operators*, *Phys. Lett.* **B 387** (1996) 483 [hep-th/9606102].

[4] N.E. Mavromatos and R.J. Szabo, *Matrix D-brane dynamics, logarithmic operators and quantization of noncommutative spacetime*, *Phys. Rev.* **D 59** (1999) 104018 [hep-th/9808124].

[5] J.R. Ellis, N.E. Mavromatos and D.V. Nanopoulos, *Dynamical formation of horizons in recoiling D-branes*, *Phys. Rev.* **D 62** (2000) 084019 [gr-qc/0006004].

[6] D. Berenstein, R. Corrado, W. Fischler, S. Paban and M. Rozali, *Virtual D-branes*, *Phys. Lett.* **B 384** (1996) 93 [hep-th/9605168];
W. Fischler, S. Paban and M. Rozali, *Collective coordinates for D-branes*, *Phys. Lett.* **B 381** (1996) 62 [hep-th/9604014].

[7] V. Gurarie, *Logarithmic operators in conformal field theory*, *Nucl. Phys.* **B 410** (1993) 535 [hep-th/9303160];
J.S. Caux, I.I. Kogan and A.M. Tsvelik, *Logarithmic operators and hidden continuous symmetry in critical disordered models*, *Nucl. Phys.* **B 466** (1996) 444 [hep-th/9511134];
M.A.I. Flohr, *On modular invariant partition functions of conformal field theories with logarithmic operators*, *Int. J. Mod. Phys.* **A 11** (1996) 4147;
M.R. Gaberdiel and H.G. Kausch, *Indecomposable fusion products*, *Nucl. Phys.* **B 477** (1996) 293;
M.R. Rahimi-Tabar, A.Aghamohammadi and M. Khorrami, *The logarithmic conformal field theories*, *Nucl. Phys.* **B 497** (1997) 555;
I.I. Kogan, A. Lewis and O.A. Soloviev, *Knizhnik-Zamolodchikov type equations for gauged WZNW models*, *Int. J. Mod. Phys.* **A 13** (1998) 1345;

I.I. Kogan and J.F. Wheater, *Boundary logarithmic conformal field theory*, *Phys. Lett.* **B 486** (2000) 353 [hep-th/0003184];
S. Kawai and J.F. Wheater, *Modular transformation and boundary states in logarithmic conformal field theory*, *Phys. Lett.* **B 508** (2001) 203 [hep-th/0103197];
M. Flohr, *Null vectors in logarithmic conformal field theory*, hep-th/0009137;
A. Lewis, *Logarithmic CFT on the boundary and the world-sheet*, hep-th/0009096;
A. Nichols and S. Sanjay, *Logarithmic operators in the SL(2,R) WZNW model*, *Nucl. Phys.* **B 597** (2001) 633 [hep-th/0007007];
A. Nichols, *Logarithmic currents in the $SU(2)_0$ WZNW model*, *Phys. Lett.* **B 516** (2001) 439 [hep-th/0102156];
S. Moghimi-Araghi, S. Rouhani and M. Saadat, *Current algebra associated with logarithmic conformal field theories*, *Lett. Math. Phys.* **55** (2001) 71 [hep-th/0012149];
Y. Ishimoto, *Boundary states in boundary logarithmic CFT*, *Nucl. Phys.* **B 619** (2001) 415 [hep-th/0103064].

[8] F. David, *Conformal field theories coupled to 2-d gravity in the conformal gauge*, *Mod. Phys. Lett.* **A 3** (1988) 1651;
J. Distler and H. Kawai, *Conformal field theory and 2-d quantum gravity or who's afraid of Joseph Liouville?*, *Nucl. Phys.* **B 321** (1989) 509.

[9] J.R. Ellis, N. E. Mavromatos and D. V. Nanopoulos, J. Chaos, Solitons and Fractals **10**, 345 C. Castro amd M.S. El Naschie eds., Elsevier Science, Pergamon 1999 and references therein; *A microscopic Liouville arrow of time*, hep-th/9805120.

[10] J.R. Ellis, N.E. Mavromatos and D.V. Nanopoulos, *Time-dependent vacuum energy induced by d-particle recoil*, *Gen. Rel. Grav.* **32** (2000) 943–958 [gr-qc/9810086];
see also: J.R. Ellis, N.E. Mavromatos and D.V. Nanopoulos, *A string scenario for inflationary cosmology*, *Mod. Phys. Lett.* **A 10** (1995) 1685 [hep-th/9503162];
G.A. Diamandis, J. Ellis, B.C. Georgalas, N.E. Mavromatos, D.V. Nanopoulos, E. Papantonopouloset al., *Irreversible time flow in a two-dimensional dilaton black hole with matter*, *Int. J. Mod. Phys.* **A 13** (1998) 4265 [gr-qc/9712051];
G.A. Diamandis, B.C. Georgalas, N.E. Mavromatos and E. Papantonopoulos, *On 'graceful exit' from inflationary phase in two-dimensional liouville string cosmology*, *Phys. Lett.* **B 461** (1999) 57 [hep-th/9903045].

[11] T. Banks and W. Fischler, *M-theory observables for cosmological space-times*, hep-th/0102077;
S. Hellerman, N. Kaloper and L. Susskind, *String theory and quintessence*, *J. High Energy Phys.* **06** (2001) 003 [hep-th/0104180];
W. Fischler, A. Kashani-Poor, R. McNees and S. Paban, *The acceleration of the universe, a challenge for string theory*, *J. High Energy Phys.* **07** (2001) 003 [hep-th/0104181].

[12] J.R. Ellis, N.E. Mavromatos and D.V. Nanopoulos, *String theory and an accelerating universe*, hep-th/0105206.

[13] C.G. Callan and I.R. Klebanov, *D-brane boundary state dynamics*, *Nucl. Phys.* **B 465** (1996) 473 [hep-th/9511173];
C. Bachas, *D-brane dynamics*, *Phys. Lett.* **B 374** (1996) 37 [hep-th/9511043];
see also C.P. Bachas, *Lectures on D-branes*, hep-th/9806199, and references therein.

[14] H. Dorn and H.J. Otto, *On t-duality for open strings in general abelian and nonabelian gauge field backgrounds*, *Phys. Lett.* **B 381** (1996) 81 [hep-th/9603186]; *Remarks on T-duality for open strings*, *Nucl. Phys.* **56** *(Proc. Suppl.)* (1997) 30;

J. Borlaf and Y. Lozano, *Aspects of t-duality in open strings*, *Nucl. Phys.* **B 480** (1996) 239 [hep-th/9607051];
Y. Lozano, *Duality and canonical transformations*, *Mod. Phys. Lett.* **A 11** (1996) 2893;
G. Amelino-Camelia and N.E. Mavromatos, *T duality for boundary-non-critical strings*, *Phys. Lett.* **B 422** (1998) 101 [hep-th/9711006].

[15] A.M. Ghezelbash and V. Karimipour, *Global conformal invariance in d dimensions and logarithmic correlation functions*, *Phys. Lett.* **B 402** (1997) 282 [hep-th/9704082];
M. Khorrami, A. Aghamohammadi and M.R. Rahimi Tabar, *Logarithmic conformal field theories with continuous weights*, *Phys. Lett.* **B 419** (1998) 179 [hep-th/9711155];
A.M. Ghezelbash, M. Khorrami and A. Aghamohammadi, *Logarithmic conformal field theories and AdS correspondence*, *Int. J. Mod. Phys.* **A 14** (1999) 2581 [hep-th/9807034].